22210

Instructions –   (1) All Questions are *Compulsory.*

(2) Answer next main Question on a new page.

(3) Illustrate your answers with neat sketches wherever necessary.

(4) Figures to the right indicate full marks.

(5) Use of Non-programmable Electronic Pocket Calculator is permissible.

(6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1.   **Attempt any FIVE of the following:**   **10**

a)   If $f(x) = 3x^2 - 5x + 7$, show that $f(-1) = 3f(1)$

b)   State whether the function $f(x) = 3x^4 + x^2 + 5 - 3\cos x + 2\sin^2 x$ is even or odd.

c)   Find $\dfrac{dy}{dx}$ if $y = e^x \cdot \sin^{-1}x$

d)   Evaluate $e^{\int 2 \cdot \log x\ dx}$

e)   Evaluate $\int \sin^2 x\ dx$

f)   Find the area under the curve $y = x^2$ from $x = 0$ to $x = 3$ with x axis.

g)   Express $z = 1 - i$ in Polar form.

Marks

2. **Attempt any THREE of the following:**   **12**

a) Find $\dfrac{dy}{dx}$ if $x^2 + y^2 = 4xy$

b) If $x = a(\theta + \sin\theta), \ y = a(1 - \cos\theta)$

   find $\dfrac{dy}{dx}$ at $\theta = \pi/2$

c) Find radius of curvature of the curve $\sqrt{x} + \sqrt{y} = 1$ at $\left(\dfrac{1}{4}, \dfrac{1}{4}\right)$

d) Find the maximum and minimum value of $x^3 - 9x^2 + 24y$

3. **Attempt any THREE of the following:**   **12**

a) Find equation of tangent and normal to the curve

   $2x^2 - xy + 3y^2 = 18$ at $(3, \ 1)$

b) Find $\dfrac{dy}{dx}$ if $y = x^x + (\sin x)^x$

c) If $y = e^{3\sec x + 4\tan x}$ find $\dfrac{dy}{dx}$

d) Evaluate $\displaystyle\int \dfrac{\sec^2 x}{(1 + \tan x)(3 + \tan x)} \, dx$

4. **Attempt any THREE of the following:**   **12**

a) Evaluate $\displaystyle\int x \tan^{-1} x \, dx$

b) Evaluate $\displaystyle\int \dfrac{dx}{4 + 5\cos x}$

c) Evaluate $\displaystyle\int \dfrac{2x^2 + 5}{(x - 1)(x + 2)(x + 3)} \, dx$

d) Evaluate $\displaystyle\int \dfrac{dx}{\sqrt{16 - 6x - x^2}}$

e) Evaluate $\displaystyle\int_0^{\pi/2} \dfrac{dx}{1 + \cot x}$

5.   **Attempt any TWO of the following:**   **12**

a) Find the area between the curves $y = x$ and $y = x^2$

b) Attempt the following:

(i)   Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \sqrt{1 + \frac{dy}{dx}}$$

(ii)   Solve

$$\frac{dy}{dx} + y\cot x = \operatorname{cosec} x$$

c) If $L\,\frac{di}{dt} = 30 \cdot \sin(10\,\pi t)$, find i in terms of t, given that L=2

and i=0 at t = 0

6.   **Attempt any TWO of the following:**   **12**

a) Attempt the following

(i)   Expresss $\dfrac{2 - \sqrt{3}\,i}{1 + i}$ in $x + iy$ form

(ii)   Find $L\{e^{-4t}\,t^2\}$

b) Find $L^{-1}\left\{\dfrac{2s^2 - 4}{(s + 1)(s - 2)(s - 3)}\right\}$

c) Solve using Laplace transform $\dfrac{dx}{dt} + 2x = e^{-t}$ given
that $x(0) = 2$

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