

22210

21718

3 Hours / 70 Marks

Seat No.

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- Instructions* –
- (1) All Questions are *Compulsory*.
 - (2) Answer each next main Question on a new page.
 - (3) Illustrate your answers with neat sketches wherever necessary.
 - (4) Figures to the right indicate full marks.
 - (5) Use of Non-programmable Electronic Pocket Calculator is permissible.
 - (6) Mobile Phone, Pager and any other Electronic Communication devices are not permissible in Examination Hall.

Marks

1. Solve any FIVE of the following:

10

- a) If $f(x) = 64^x + \log_3 x$, find $f\left(\frac{1}{3}\right)$
- b) If $f(x) = \sin x$, show that $f(3x) = 3f(x) - 4f^3(x)$
- c) Find $\frac{dy}{dx}$ if $y = e^x \sin^{-1} x$
- d) Evaluate : $\int x(x-1)^2 dx$
- e) Evaluate : $\int \sin^2 2x dx$
- f) Find the area bounded by the curve $y = x^2$, x - axis and ordinates $x = 0$ to $x = 3$.
- g) Express $z = \frac{1-i}{1+i}$ in $a+ib$ form, where $i = \sqrt{-1}$ and a, b are real numbers.

P.T.O.

2. Attempt any THREE of the following:

12

- a) If $13x^2 + 2x^2y + y^3 = 1$, find $\frac{dy}{dx}$ at $(1, -2)$
- b) If $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$, find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{2}$
- c) The rate of working of an engine is given by the expression $10V + \frac{4000}{V}$, where V is the speed of the engine. Find the speed at which the rate of working is the least.
- d) A telegraph wire hangs in the form of a curve $y = 2 \sin x - \sin 2x$. Find the radius of curvature of the wire at the point $x = \frac{\pi}{2}$

3. Solve any THREE of the following:

12

- a) Find the equation of the tangent to the curve $y = 9x^2 - 12x + 7$ which is parallel to the x -axis.
- b) Find $\frac{dy}{dx}$ if $y = \log\left(\frac{\sin x}{1 + \cos x}\right)$
- c) If $x^y = e^{x-y}$, then prove that $\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$
- d) Evaluate : $\int \frac{\cos x}{1 + \sin^2 x} dx$

4. Solve any THREE of the following:

12

- a) Evaluate : $\int \frac{\log x}{x(2 + \log x)} \cdot \frac{dx}{(3 + \log x)}$
- b) Evaluate : $\int \frac{dx}{3 - 2 \sin x}$
- c) Evaluate : $\int \frac{x \sin^{-1} x}{\sqrt{1 - x^2}} dx$

- d) Evaluate : $\int \frac{x+1}{x^2(x-2)} dx$
- e) Evaluate : $\int_1^3 \frac{\sqrt[3]{x+5}}{\sqrt[3]{x+5} + \sqrt[3]{9-x}} dx$

5. Solve any TWO of the following: **12**

- a) Find the area enclosed between the parabola $y = x^2$ and the line $y = 4$.

b) **Attempt the following:**

- (i) Find the order and degree of the differential equation

$$\frac{d^2y}{dx^2} = \left(y + \frac{dy}{dx} \right)^{3/2} \quad 2$$

- (ii) Solve : $x \frac{dy}{dx} - y = x^2$ 4

- c) The current ' i ' is given by $L \frac{di}{dt} = 30 \sin(10\pi t)$, where L is inductance and t is time. Find ' i ' in terms of t , given that $L = 2$ and $i = 0$ at $t = 0$.

6. Solve any TWO of the following: **12**

a) **Attempt the following:**

- (i) If $z_1 = -3 + 4i$, $z_2 = 5 - 3i$ express $\frac{z_1}{z_2}$ in $x + iy$ form.

- (ii) Find $L \{e^{-3t} \sin 2t\}$

b) Find $L^{-1} \left\{ \frac{3s+1}{(s-1)(s^2+1)} \right\}$

- c) Solve the differential equation using Laplace transform.

$$L \frac{di}{dt} + R i = V, \quad i(0) = 0$$
